2013 State NMJAS Paper Competition Winner

Amanda Mercer

Tautochromo Time

Los Alamos High School, Los Alamos, New Mexico

Problem: The tautochrone problem is one of the earliest to be solved using calculus. The problem is to find a curve down which a point placed anywhere will slide to the bottom in the same amount of time. Christian Huygens, a clockmaker, first solved this problem in 1673, and the curve he discovered is a cycloid. I want to confirm this discovery using modern calculus, numerical methods, and physical measurements of a cycloid-path pendulum.

Hypothesis: The period of a cycloid pendulum is independent of amplitude, so it is a tautochrone.

Procedure: 1) Use modern calculus methods to find the period of a cycloid pendulum and demonstrate that this period is independent of amplitude. 2) Use numerical methods to find a tautochrone shape without prior assumption, then compare to a cycloid. 3) Construct a physical cycloid pendulum and test whether the period is independent of amplitude. Try the same for a simple circular pendulum.

Data/Results: Using only first-semester high school calculus, I was able to recreate the proof that a cycloid is a tautochrone. The numerical method for finding a tautochrone curve also produced a cycloid shape. My physical measurements confirmed my hypothesis: The period of the cycloid pendulum varied by only plus or minus 0.045% over the full range of amplitudes. The period of the circular pendulum varied by plus or minus 2.8% over the same range.

Conclusions: A cycloid pendulum is a tautochrone; unlike the simple circular pendulum, its period is independent of amplitude.